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Radiative Corrections and the Standard Model of Elementary Particles¹

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Abstract

This presentation includes an introductory discussion of the unification of fundamental forces, properties of the elementary particles, Quantum Electrodynamics, the transition from Quantum Electrodynamics and Weak Interactions to Electroweak Physics, Radiative Corrections in the Standard Model of Particle Physics, and some critical unsolved problems.

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1 Introduction

The XIX century witnessed two great unifications in Physics. During the first decades of the century, major progress was achieved in understanding the interdependence of electric and magnetic phenomena by the work of great physicists like Ampère, Biot and Savart, Oersted, Faraday, and others. This process culminated in Maxwell's theory, in which two separate areas of Physics, Electricity and Magnetism, were unified in a single discipline, Classical Electromagnetism.

But Maxwell went beyond: studying the mathematical solutions of his equations, he reached the conclusion that they predict the existence of electromagnetic waves that propagate in vacuum with the speed of light. Their presence was soon confirmed experimentally by Hertz, and this discovery ushered the era of modern telecommunications and technology. Maxwell then proposed that light is an electromagnetic wave in a restricted range of frequencies and, in a single stroke, achieved a second great unification, that of Optics and Electromagnetism.

Maxwell's treatise on Classical Electromagnetism was published in 1874. Considered by many the most important theoretical physicist of the XIX century, he died in 1879, at the early age of 47. The same year of 1879 witnessed the birth of his great successor, Albert Einstein. (Curiously enough, Newton was born in 1642, the year of Galileo's death). In his Special Theory of Relativity, Einstein addressed some profound contradictions between Classical Electromagnetism and Newtonian Mechanics. He left untouched Maxwell's theory, but altered Newtonian Mechanics and, in so doing, revolutionized our understanding of space and time.

Approximately one hundred years later, a partial unification of three of the four fundamental forces of nature, the electromagnetic, weak and strong interactions, was achieved. This led to the emergence of the Standard Model of Elementary Particles (1967-74), proposed originally by Weinberg, Salam, and Glashow, with very important contributions from other physicists.

2 Brief Synopsis about Elementary Particles

At present, physicists distinguish four fundamental forces in nature. They manifest themselves as interactions between the fundamental particles.

- 1) Strong Interactions: responsible, for example, for the binding of neu-

trons and protons in the atomic nuclei. They are a crucial factor in nuclear physics. 2) Electromagnetic Interactions: responsible, for example, for the binding of electrons and atomic nuclei to form atoms. They play a fundamental role in atomic physics, chemistry, and biology. 3) Weak Interactions: responsible for radioactive β decays (for instance, $n \rightarrow p + e^- + \bar{\nu}_e$), and for many decay processes involving the elementary particles. 4) Gravitational Interactions: responsible, for example, for the dynamics of the solar system. They are a crucial factor in the large scale structure of the universe.

These interactions are transmitted by certain fundamental particles that act as mediators. For example, the electromagnetic interaction between two electrons is described by the Feynman diagram

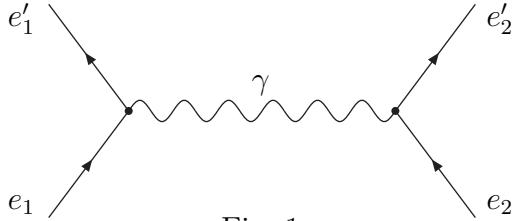
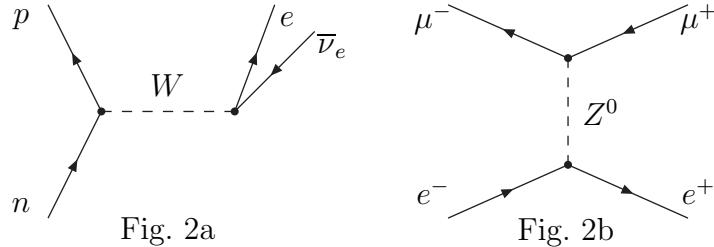


Fig. 1

Here, an electron in the quantum state e_1 propagates in space-time and emits a virtual photon γ that also propagates in space-time and interacts with the second electron. As a consequence of the interactions, the two electrons change their quantum states from e_1 and e_2 to e'_1 and e'_2 , respectively. The photon has zero mass and spin 1.

The weak interactions are mediated by three very massive intermediate bosons: W^\pm , Z^0 . Examples are



The W and Z bosons are about 86 and 97 times more massive than

the proton, respectively, and have spin 1. Discovered in the decade 1980 to 1990, they were studied in great detail in major laboratories: CERN (Geneva, Switzerland), SLAC (Stanford, California), Fermilab (ca. Chicago, Illinois). The present values of their masses are $M_W = 80.426 \pm 0.034 \text{ GeV}/c^2$, $M_Z = 91.1875 \pm 0.0021 \text{ GeV}/c^2$. ($\text{GeV}/c^2 \approx 1.8 \times 10^{-27} \text{ Kg}$). (In order to achieve the great precision in the measurement of M_Z , physicists at CERN had to take into account the schedule of electric trains in the vicinity of Geneva and the gravitational effects of the moon!).

The present theory of strong interactions is called Quantum Chromodynamics. The mediators are eight gluons. Like the photon, they have zero mass and spin 1. They mediate the strong interactions between the quarks.

The fundamental matter fields are the leptons (which are not affected by the strong interactions) and the quarks (which are). They have spin 1/2 and appear in three generations, shown below.

$$\begin{aligned}
\text{1st Generation} & \left\{ \begin{array}{ll} \nu_e & < 3 \text{ eV}/c^2 \\ e & \approx 0.51 \text{ MeV}/c^2 \\ u \text{ (up)} & (\text{several}) \text{ MeV}/c^2 \\ d \text{ (down)} & (\text{several}) \text{ MeV}/c^2 \end{array} \right. \\
\\
\text{2nd Generation} & \left\{ \begin{array}{ll} \nu_\mu & < 0.19 \text{ MeV}/c^2 \\ \mu \text{ (muon)} & \approx 106 \text{ MeV}/c^2 \\ c \text{ (charm)} & \approx 1.2 \text{ GeV}/c^2 \\ s \text{ (strange)} & \approx 120 \text{ MeV}/c^2 \end{array} \right. \\
\\
\text{3rd Generation} & \left\{ \begin{array}{ll} \nu_\tau & < 18 \text{ MeV}/c^2 \\ \tau \text{ (\tau-lepton)} & \approx 1.78 \text{ GeV}/c^2 \\ t \text{ (top)} & = (174.3 \pm 5.1) \text{ GeV}/c^2 \\ b \text{ (bottom)} & \approx 4.3 \text{ GeV}/c^2 \end{array} \right. \\
\\
\text{Intermediate Bosons} & \left\{ \begin{array}{ll} \gamma & 0 \\ W^\pm & = (80.426 \pm 0.034) \text{ GeV}/c^2 \\ Z^0 & = (91.1875 \pm 0.0021) \text{ GeV}/c^2 \\ g \text{ (gluons)} & 0 \end{array} \right. \\
\\
\text{Higgs Boson} & H > 115 \text{ GeV}/c^2
\end{aligned}$$

The charged leptons (e, μ, τ) have charge -1 in units of the proton charge, while the accompanying neutrinos (ν_e, ν_μ, ν_τ) are neutral. The quarks come in six flavors (u, d, c, s, t, b). The u, c, t quarks have charge $2/3$ while d, s, b have charge $-1/3$. Quarks and gluons are also endowed with another attribute called “color” that is associated with their strong interactions. On the right side we have indicated the approximate masses ($\text{MeV}/c^2 \approx 1.8 \times 10^{-30} \text{ kg}$) or upper and lower bounds. Corresponding to each lepton or quark, there is an antiparticle with the same mass and opposite charge.

In the chart we have also included the intermediate bosons discussed before and the Higgs boson H of spin 0, a fundamental particle of the SM that so far has not been discovered. Its interactions with the other particles play a crucial role in the generation of their masses.

Photons and neutrinos are very abundant. In each cm^3 of intergalactic space there is an average of 412 photons and 112 neutrinos of each species. The upper bounds on the neutrino masses are so small that, until recently, the possibility existed that they may be massless. However, in the last three years, very strong evidence has been found that they oscillate among themselves. For example $\nu_e \rightarrow \nu_\mu, \nu_\mu \rightarrow \nu_\tau$, which is only possible if they have mass. The study of neutrinos produced by the sun, and of atmospheric neutrinos, has led to $|m_{\nu_\mu}^2 - m_{\nu_e}^2| \approx 5 \times 10^{-5} \text{ eV}^2$ and $|m_{\nu_\tau}^2 - m_{\nu_\mu}^2| \approx 3 \times 10^{-3} \text{ eV}^2$, respectively.

If we assume that $m_{\nu_\mu}^2 \gg m_{\nu_e}^2$, we find $m_{\nu_\mu} \approx 7 \times 10^{-3} \text{ eV}$, much smaller than the present upper bound.

3 Quantum Electrodynamics

In the first three decades of the XX century two great revolutions took place in physics:

Special and General Relativity, developed by Einstein, and Quantum Mechanics, associated with a large group of extraordinary physicists (Schrödinger, Heisenberg, Bohr, Pauli, Born, Dirac, Einstein, Planck, ...).

The combination of Electromagnetism with Quantum Mechanics and Special Relativity culminated in a very deep and successful theory called Quantum Electrodynamics (QED) (Feynman, Schwinger, Tomonaga, Dyson, Bethe, ...).

As I mentioned before, in first approximation the interaction between two electrons can be described by Fig.1. Similarly, the interaction of an electron

with an external source, such as an atomic nucleus, is represented by

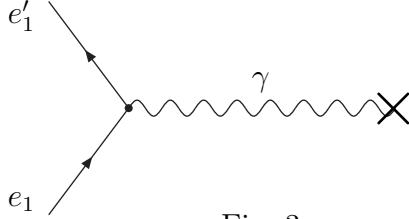


Fig. 3

Here, the \times represents the atomic nucleus that emits a virtual photon, which in turn interacts with the electron. Fig. 3 describes the scattering of the electron by its electromagnetic interaction with the nucleus. QED leads to the conclusion that there are subtle corrections to this process that can be evaluated systematically. An example is

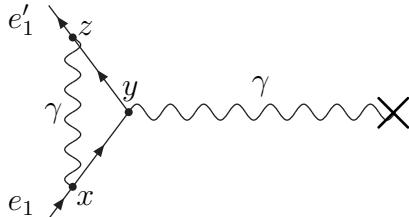


Fig. 4

Here, an electron propagates in space-time, emits a virtual photon at a space-time point x , at y absorbs the photon emitted by the nucleus , and at z absorbs the photon it has previously emitted. Fig. 4 is called a Radiative Correction or a Quantum Correction to the basic scattering process of Fig. 3. In QED these corrections are evaluated as a series in powers of the fine structure constant

$$\alpha = \frac{e^2}{\hbar c} = \frac{1}{137.03599877(40)},$$

where e is the electron charge, \hbar the Planck's constant, c the speed of light and (40) indicates the experimental error that resides in the last two digits. Each power of α corresponds to a loop in the Feynman diagram. For instance, Fig. 4 contains one loop and leads to a correction proportional to α . There

are other Feynman diagrams in which, for example, the electron emits and absorbs n virtual photons leading to n loops and a correction proportional to α^n .

The calculation of the QED corrections leads to predictions of very high precision. For example, an electron possesses a magnetic moment $\vec{m} = \frac{ge\hbar}{2m_e c} \vec{S}$, where \vec{S} is its spin and m_e its mass, an attribute that governs its interaction with magnetic fields. According to Dirac's theory of the electron, which is a relativistic generalization of quantum mechanics, $g = 2$. Schwinger showed that the QED radiative corrections associated with Fig. 4 alter this result leading to $a_e \equiv (g - 2)/2 = \alpha/2\pi$. By now the corrections have been computed through $\mathcal{O}(\alpha^4)$ and the experiments carried out with very high accuracy both for the electron and the μ -meson (muon). The experimental and theoretical values are given below

$$\begin{aligned} a_{e^-}^{exp} &= 1.1596521884(43) \times 10^{-3} \\ a_{e^+}^{exp} &= 1.1596521879(43) \times 10^{-3} \\ a_e^{th} &= 1.1596521640(160) \times 10^{-3} \\ \\ a_\mu^{exp} &= 1.16592030(80) \times 10^{-3} \\ a_\mu^{th} &= 1.16591693(78) \times 10^{-3} \\ a_\mu^{th} &= 1.16591890(71) \times 10^{-3} \end{aligned}$$

In the case of the muon, which is of great current interest, we have presented two recent calculations in which subtle strong interaction effects have been evaluated by different methods. At the present level of precision, these two calculations differ in the last digits, but it is expected that the origin of this discrepancy will be understood better in the near future.

It is clear from the above numbers that QED is being tested with extraordinary precision. For instance, in the electron case we are dealing with 11 significant figures, with the error placed in the last two or three digits!

4 Transition from QED and Weak Interactions to Electroweak Physics

The fact that the radiative corrections described in Section 3 can be evaluated consistently is due to an important property of QED, namely it is

a renormalizable theory. In such theories, divergent contributions to the mathematical expressions associated with Feynman diagrams are eliminated as unobservable contributions to the fundamental parameters. For instance, in QED the divergent parts in the evaluation of the diagrams are absorbed as unobservable contributions to the mass and charge of the electron. This process of removing the divergent contributions is called Renormalization.

The original theory of weak interactions, due to the great theoretical and experimental physicist Enrico Fermi (1934), had remarkable success in describing and relating a large number of phenomena and experimental results. However, it was not renormalizable. There were several attempts to create a renormalizable theory of weak interactions, but in general they were not successful. Finally, in the period 1967-1974 the Standard Model (SM) emerged with the aim of achieving a unification of the weak, electromagnetic and strong interactions. Soon afterwards, the work of 't Hooft, Veltman, Ben Lee, Zinn-Justin, Becchi, Rouet, Stora, ... showed that the SM is renormalizable. The original analysis of 't Hooft and Veltman was constructed on the basis of a technique, called Dimensional Regularization (DR), used to give mathematical meaning to the divergent integrals associated with Feynman diagrams. Curiously enough, DR was invented, almost simultaneously, in three different places: by Bollini and Giambiagi in Argentina, 't Hooft and Veltman in the Netherlands, and Ashmore in Italy.

The renormalizability of the SM opened the possibility to study the Radiative or Quantum Corrections in a consistent manner. As the theory mixes the electromagnetic and weak contributions, we call them Electroweak Corrections (EWC).

My first tentative steps in the study of the SM coincided with an extremely fruitful visit to Argentina (January-August of 1972). Soon after I arrived, I had a memorable conversation with Bollini and Giambiagi, who explained to me the idea of DR. Soon afterwards, we learned from Victor Alessandrini, who arrived from Europe, that 't Hooft and Veltman had also proposed DR in the very important context of gauge theories. During that visit I gave classes on the SM at the University of La Plata, that were very well attended by people from La Plata and Buenos Aires, and carried out my first research work in this area. With H. Fanchiotti and H. Girotti we discussed the cancellation of ultraviolet divergencies in the unitary-gauge treatment of photon-photon scattering, and with Bollini and Giambiagi the cancellation of ultraviolet divergencies in natural relations of the SM.

My principal area of interest since that time has been the study of the

EWC to important processes, with the aim to establish a close contact between theory and high precision experiments.

The desideratum of these studies are

- i) To verify the SM at the level of its quantum corrections, attempting to follow the great example of QED.
- ii) To search for discrepancies between theory and experiment or indications that may signal the presence of new physics beyond the SM.

These are the fundamental objectives of what is now known as Precision Electroweak Physics.

5 Electroweak Corrections in the Standard Model

I will give some illustrative examples.

5.1 Unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix

The interactions of the W^\pm intermediate bosons with quarks are governed by a matrix

$$V = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix},$$

where, for example, V_{ud} refers to the coupling with the u, d quarks. According to the theory, an important property is that V is a unitary matrix, which implies $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$ with analogous equalities for the other rows and columns of V . $|V_{ud}|$ is determined by comparing the lifetimes of the muon and β decays, which are known with great experimental precision. On the theoretical side one needs the EWC to both processes. This problem had already been studied in the Fermi V-A theory that preceded the SM (T. Kinoshita, A. Sirlin, S.M. Berman (1958-59)). At that time we found a great theoretical difficulty: while the EWC to the muon lifetime were finite, those involving β decay were divergent. This was related to the fact that the Fermi theory of weak interactions is not renormalizable.

When the renormalizability of the SM was recognized in the early seventies, I thought it was urgent to re-examine the problem in the light of the new theory. In 1974 I obtained the answer in a simplified version of the SM, ignoring the effect of the strong interactions (SI). During 1974-78 I extended the analysis to the full-fledged SM, including the effect of the SI. The EWC are now finite (as expected in a renormalizable theory) and of sizable magnitude. They are dominated by a large logarithmic contribution $(3\alpha/2\pi) \ln(M_Z/2E_m) \sim 3.4\%$, where E_m is the maximum energy of the electron (or positron) in β decay. It turns out that such large contribution was indeed required phenomenologically to satisfy the unitarity of the V matrix. For me, that was the “smoking gun” of the SM at the level of the quantum corrections!

5.2 Prediction of the M_W and the M_Z masses

In 1979-81, William Marciano (who was a former student of mine) and I thought that experimental physicists would attempt to discover the W^\pm and Z^0 intermediate bosons, predicted by the SM, and measure their masses M_W and M_Z . It seemed a good idea to study, at the level of the EWC, the relations between M_W , M_Z and G_F , α , as well as other fundamental parameters of the theory such as the fermion masses, generically denoted by M_f , and the mass M_H of the Higgs boson. Here $G_F = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2}$ is a very important constant that measures the magnitude of the weak interactions in the Fermi theory. We reached the conclusion that in order to predict M_W and M_Z it would be necessary to evaluate the EWC to a number of different processes mediated by the W^\pm and Z^0 . As this seemed to be a difficult task and the theoretical formulations at that time were very complicated, I thought the first step should be the development of a simple method to renormalize the Electroweak Sector of the SM (A. Sirlin, Physical Review D22, 971 (1980)). This approach, with subsequent important contributions from other physicists, is presently called the “on-shell scheme of renormalization”. Applying this scheme to muon decay, one finds

$$\sin^2 \theta_W \cos^2 \theta_W = \frac{A^2}{M_Z^2(1 - \Delta r)},$$

where $A^2 = \pi\alpha/(\sqrt{2}G_F)$, $\sin^2 \theta_W = 1 - M_W^2/M_Z^2$ and Δr is the corresponding EWC. It has a complex structure and depends on several important parameters, in particular M_H , M_W , M_Z and the top quark mass M_t . In combination

with experiments involving neutrino collisions with atomic nuclei at high energies, the previous relations permitted to gradually improve the predictions of M_W and M_Z .

Around 1989, the great accelerators LEP at CERN and SLC at SLAC started operations and Fermilab began the precision measurements of M_W . As mentioned in Sect. 1, LEP soon determined M_Z with great accuracy, and this led to a change in strategy: α , G_F and M_Z were adopted as the basic input parameters, and a major effort was done to study the Z^0 resonance in processes of the type of Fig. 2b. In particular, several on resonance asymmetries and widths were measured with precision. At present we know α , G_F and M_Z with uncertainties of $\delta\alpha = \pm 0.0037$ ppm, $\delta G_F = \pm 9$ ppm, $\delta M_Z = \pm 23$ ppm, where ppm is an abbreviation for “parts per million”.

In general, the experimental precision of the other observables is of the order of 0.1% and this makes necessary to include the EWC in the theoretical predictions.

Thus, theorists working in this area were lucky: experimental physics in the great accelerators moved in the direction of high precision, where the study of the EWC is particularly important!

5.3 M_t Prediction

A very interesting example of the successful interplay between theory and experiment was the M_t prediction and its subsequent measurement. Before 1995, the top quark could not be produced directly, but it was possible to estimate its mass because of its contributions to the EWC. In Nov. 1994, a global analysis of the comparison between the SM and the experiments led to the indirect determination

$$M_t = 178 \pm 11^{+18}_{-19} \text{ GeV}/c^2,$$

where the central value corresponds to $M_H = 300 \text{ GeV}/c^2$, the first error is experimental and the second shift assumes $M_H = 65 \text{ GeV}/c^2$ ($-19 \text{ GeV}/c^2$) or $M_H = 10^3 \text{ GeV}/c^2$ ($+18 \text{ GeV}/c^2$).

Finally, with increasing energy and luminosity the top quark was produced at Fermilab and its mass measured. Its present value is

$$M_t = 174.3 \pm 5.1 \text{ GeV}/c^2,$$

very close to the prediction!

The possibility of this successful prediction is due to the fact that Δr and other important EWC depend quadratically on M_t , i.e. they contain contributions proportional to M_t^2 and are, therefore, sensitive functions of M_t .

5.4 The Higgs Boson

This is a fundamental particle in the SM that, as mentioned in Section 2, so far has not been found. Its interactions provide the mass of all the other particles: intermediate vector bosons, leptons and quarks. The direct search indicates that its mass $M_H \geq 114.4 \text{ GeV}/c^2$ at the 95 % confidence level. With M_t known experimentally, an important problem is the estimation of M_H by studying its effect on the EWC. This is much more difficult than the M_t prediction because the EWC depend only logarithmically on M_H , and are therefore much less sensitive to this basic parameter. One of the most important factors in constraining M_H is the EWC Δr mentioned in Subsection 5.2. The comparison between the theory and the current measurements of the various observables leads to the conclusion that $M_H < 211 \text{ GeV}/c^2$ at the 95 % confidence level. We have therefore a limited band $114.4 \text{ GeV}/c^2 \lesssim M_H \lesssim 211 \text{ GeV}/c^2$, that will be explored at Fermilab and the new accelerator LHC (large hadron collider) under construction at CERN.

5.5 Supersymmetry

One of the most interesting theoretical possibilities that involves new physics beyond the SM is Supersymmetry (SUSY). It is a theory that, among many other features, predicts that every boson (particle with integer spin) has a fermion partner (particle of half-integer spin), and viceversa. In its simplest form SUSY leads to an extension of the SM called MSSM (minimal supersymmetric model). One of its most important predictions is that there are five Higgs bosons and that the lightest one satisfies $M_H \lesssim 130 \text{ GeV}/c^2$. The EWC have been also studied in great detail in the MSSM framework and play a crucial role in the derivation of the M_H upper bound.

On the other hand, supersymmetric partners of the usual elementary particles have not been discovered thus far.

5.6 Grand Unification

The SM is invariant under certain mathematical transformations, called gauge transformations, which are associated with a symmetry group $SU(2) \times U(1) \times SU(3)$. The first two factor groups describe the symmetry properties of the Electroweak Sector of the theory, while the third involves the Strong Interactions (QCD). The three factor groups are characterized by parameters $g(\mu)$, $g'(\mu)$, $g_s(\mu)$ that determine the magnitude of the interactions at the energy scale μ of the phenomena under consideration. An attractive idea is the possibility that the three parameters are unified at a high μ scale, where the symmetry is described by a single factor group, such as $SU(5)$ or $SO(10)$. This possibility is called Grand Unification. It has been shown that the three lines defined by $g(\mu)$, $g'(\mu)$ and $g_s(\mu)$ as functions of μ , in fact intersect at $\mu \simeq 10^{16} \text{ GeV}/c^2$ in the presence of SUSY, but not in its absence. This is one of the reasons that make the MSSM theoretically attractive. In this analysis of Grand Unification the EWC play also a very important role because they permit to obtain accurate values for $g(\mu)$, $g'(\mu)$ and $g_s(\mu)$ at the energy scale of the current experiments, which are important inputs in the calculations, and also govern the evolution of these parameters as functions of μ .

A major prediction of Grand Unified Theories, which has not been verified yet, is that the proton is unstable, albeit with an extremely long lifetime.

6 Conclusions

- i) The SM is a theory that describes with high precision a multitude of phenomena from the atomic energy scale ($\simeq 10 \text{ eV}$) up to $\simeq 100 \text{ GeV}$ (ten orders of magnitude!)
- ii) It is a renormalizable theory so that its quantum corrections can be evaluated systematically, and this permits to compare the theoretical predictions with high precision experiments.
- iii) This comparison has generally been very successful in demonstrating
 - a) that the SM is correct at the 0.1% level (assuming that the Higgs boson will be found at a consistent mass scale), verifying the principle of gauge invariance, the symmetry group $SU(2) \times U(1) \times SU(3)$ and the representations of this group assigned to the various particles.

- b) that the EWC and QCD corrections are essentially correct, verifying the validity of renormalizable gauge theories.
- iv) The comparison has also permitted
 - c) to determine important parameters such as $\sin^2 \theta_W$, to predict M_t and estimate M_H .
 - d) to sharply restrict possible new physics beyond the SM to be of a type in which heavy new particles decouple at energies much lower than their masses, such as supersymmetry.

A major unsolved problem is the unification of gravity with the other three forces of nature and, more generally, the harmonious combination of the two great revolutionary theories of the XX century, namely general relativity and quantum theory. Many theorists believe that string theory, in which elementary particles are regarded as excitations of fundamental strings, rather than point structures, offers the most promising paradigm to achieve these critical aims.

Acknowledgment

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